

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 1	Daily Tutorial Sheet 2
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16.(3) $|x + 4| = 3x - 2$

C.P. = -4

$$\begin{array}{c} \text{(i)} \quad \text{(ii)} \\ \hline (-) \quad -4 \quad (+) \end{array}$$

Case (i) $x \leq -4$: $-x - 4 = 3x - 2 \Rightarrow x = -\frac{1}{2}$ (not possible)

Case (ii) $x > -4$: $x + 4 = 3x - 2 \Rightarrow 2x = 6 \Rightarrow x = 3$

17. $\left(\frac{-1 \pm \sqrt{21}}{2} \right)$

$|x^2 + x| - 5 = 0$

C.P. 0, -1

$$\begin{array}{ccccccc} & & | & & | & & \\ (+) & -1 & & (-) & 0 & & (+) \end{array}$$

Case 1: $x \leq -1$

$$x^2 + x - 5 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{21}}{2} \Rightarrow x = \frac{\sqrt{21} - 1}{2}, \frac{-\sqrt{21} - 1}{2}$$

$$\therefore x = -\frac{\sqrt{21} - 1}{2} \text{ (as } x \leq -1)$$

Case 2: $-1 < x \leq 0$

$$\therefore -x^2 - x - 5 = 0$$

$$x^2 + x + 5 = 0 \text{ no solution}$$

Case 3: $x > 0$

$$x^2 + x - 5 = 0 \Rightarrow x = \frac{\sqrt{21} - 1}{2}, \frac{-\sqrt{21} - 1}{2}$$

$$\therefore x = \frac{\sqrt{21} - 1}{2} \therefore [\text{as } x > 0] \quad \therefore x = \frac{\sqrt{21} - 1}{2}, \frac{-\sqrt{21} - 1}{2}$$

Alternate Solution:

$$|x^2 + x| = 5 \text{ iff } x^2 + x = \pm 5$$

$$x^2 + x = -5 \text{ has no solution.}$$

$$x^2 + x = 5 \Rightarrow x = \frac{-1 \pm \sqrt{21}}{2}$$

18. $2\sqrt{2}, 2(1 + \sqrt{3})$

$|x^2 - 2x - 8| = 2x$

$$\text{C.P.} = x = 4, -2$$

$$\text{Case 1: } x \leq -2$$

$$x^2 - 24 - 8 = 2x \Rightarrow x^2 - 4x - 8 = 0 \Rightarrow x = 2 \pm 2\sqrt{3} \text{ (not possible)}$$

$$\text{Case 2: } -2 < x \leq 4$$

$$-x^2 + 2x + 8 = 2x$$

$$\Rightarrow x^2 = 8 \Rightarrow x = \pm 2\sqrt{2}$$

$$\therefore x = 2\sqrt{2}$$

$$\text{Case 3: } x > 4$$

$$\therefore x = 2 \pm 2\sqrt{3} \Rightarrow x = 2 + 2\sqrt{3}$$

$$\therefore x = 2\sqrt{2}, \quad 2 + 2\sqrt{3}$$

$$19. \quad |x^2 - x - 6| = x + 2$$

$$\text{C.P.} = -2, 3$$

$$(x-3)(x+2) = 0$$

$$x = 3, -2$$

$$\text{Case 1: } x \leq -2$$

$$x^2 - x - 6 = x + 2$$

$$\Rightarrow x^2 - 2x - 8 = 0 \quad \therefore x = \frac{2 \pm \sqrt{4+32}}{2} = 4, -2 \quad \therefore x = -2$$

$$\text{Case 2: } -2 < x \leq 3$$

$$-x^2 + x + 6 = x + 2$$

$$\Rightarrow x^2 = 4$$

$$x = \pm 2 \quad \therefore x = 2$$

$$\text{Case 3: } x > 3$$

$$\therefore x = 4, -2 \Rightarrow x = 4 \quad \therefore x = -2, 2, 4$$

Alternate solution:

$$|x+2| |x-3| = x+2$$

Clearly, $x = -2$ is a solution.

$$\text{Case - I: } x+2 < 0$$

$$\Rightarrow |x-3| = -1 \quad \{\text{Impossible}\}$$

$$\text{Case - II: } x+2 > 0$$

$$|x-3| = 1$$

$$\Rightarrow x = 2, 4$$

$$20. \quad \text{Let } |x+1| = y \quad \therefore 2y^2 - y - 3 = 0 \Rightarrow 2y^2 - 3y + 2y - 3 = 0$$

$$\Rightarrow (2y-3)(y+1) = 0 \Rightarrow |x+1| = \frac{3}{2}, -1 \Rightarrow x+1 = \pm \frac{3}{2} \quad \therefore -4 = \frac{1}{2}, -\frac{5}{2}$$

$$21. [2, \infty)$$

$$\text{CP} = 0, 1, 2$$

$$\text{Case 1: } x \leq 0$$

$$-x - 2x + 2 + 3x - 6 = 4 \Rightarrow -4 = 4 \quad (\text{No solution})$$

$$\text{Case 2: } 0 < x \leq 1$$

$$x - 2x + 2 + 3x - 6 = 4 \Rightarrow 2x = 8$$

$$x = 4 \text{ (not possible)}$$

Case 3: $1 < x \leq 2$

$$x + 2x - 2 + 3x - 6 = 4 \Rightarrow 6x = 12$$

$$x = 2$$

Case 4: $x > 2$

$$x + 2x - 2 - 3x + 6 = 4 \Rightarrow 4 = 4$$

$$x \in R$$

But $x > 2$

Soln: $x \geq 2$

22.(1) CP = 3, 1 $\frac{1}{1} \quad \frac{2}{3}$

Case 1: $x \leq 1$

$$2(3-x) + 3(1-x) = 4 \Rightarrow 9 - 5x = 4 \Rightarrow 5x = 5$$

$$x = 1$$

Case 2: $1 < x \leq 3$

$$2(3-x) + 3(x-1) = 4$$

$$\Rightarrow 6 + x - 3 = 4 \Rightarrow x = 1 \text{ (not possible)}$$

Case 3: $x > 3$

$$2(x-3) + 3(x-1) = 4 \Rightarrow 5x = 13$$

$$x = \frac{13}{5}$$

But $x > 3$

$$\therefore x = \frac{13}{5} \text{ (not possible)} \quad \therefore x = 1$$

23. CP = -1, 0, 1 $\frac{1}{-1} \quad \frac{2}{0} \quad \frac{3}{1} \quad \frac{4}{1}$

Case 1: $x \leq -1$

$$\therefore x^2 - x - x - 1 = 5 \Rightarrow x^2 - 2x - 6 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4+24}}{2} = 1 \pm \sqrt{7} \therefore x = 1 - \sqrt{7}$$

Case 2: $-1 < x \leq 0$

$$x^2 - x + x + 1 = 5 \Rightarrow x^2 = 4$$

$$x = \pm 2 \text{ (not possible)}$$

Case 3: $0 < x \leq 1$

$$-x^2 + x + x + 1 = 5 \Rightarrow x^2 - 2x + 4 = 0$$

$D < 0$ no real roots.

Case 4: $x > 1$

$$x^2 - x + x + 1 = 5 \Rightarrow x = \pm 2 \therefore x = 2$$

Soln.: $x = (1 - \sqrt{7}), 2$

24.(2,1) CP = $\frac{3}{2}$

Case 1: $x \geq \frac{3}{2}$

$$x^2 + x + 1 + 2x - 3 = 4x \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = 2, -1 \therefore x = 2$$

Case 2: $x < \frac{3}{2} \therefore x^2 + x + 1 - 2x + 3 = 4x \Rightarrow x^2 + x - 1 - 2x + 3 = 4x \Rightarrow x^2 - 5x + 4 = 0$

$$\Rightarrow (x-1)(x-4) = 0$$

$$x = 1, 4$$

Ans: $x = 1, 2$

25. $2^x \geq 0$
 $\Rightarrow 2x+1 + |2x-1| = 2$

C.P. $2^x = 1 \Rightarrow x = 0$

Case - I: $x \leq 0$

$$2^x + 1 + 1 - 2^x = 2$$

Always True. Hence, $x \leq 0$

Case - II: $x > 0$

$$2^x + 1 + 2^x - 1 = 2$$

$$\Rightarrow x = 0 \text{ (Not possible).}$$

26. We have $\frac{1}{x} - \frac{2}{x-2} < 0 \Rightarrow \frac{x-2-2x}{x(x-2)} < 0 \Rightarrow \frac{-x-2}{x(x-2)} < 0$

$$\begin{array}{c} + \quad + \\ \hline - \quad -2 \quad 0 \quad - \quad 2 \\ \hline x+2 \\ \hline x(x-2) \end{array}$$

Thus, the solution set is $(-2, 0) \cup (2, \infty)$.

27. $\frac{x+4}{x-2} - \frac{2}{x+1} < 0 \Rightarrow \frac{x^2+5x+4-2x+4}{(x-2)(x+1)} < 0 \Rightarrow \frac{x^2+3x+8}{(x-2)(x+1)} < 0$

$$\begin{array}{c} \hline (+) \quad -1 \quad (-) \quad 2 \quad (+) \\ \hline \end{array}$$

$$\Rightarrow \frac{1}{(x-2)(x+1)} < 0 \text{ (}\because x^2+3x+8 > 0 \text{ for all } x) \Rightarrow \text{Hence the solution set is } (-1, 2)$$

28. ϕ

Since $2x^2 - 3x + 5 = 0$ does not have real roots, the solution to the inequation is either all real x or no real x .

Now $a = 2 > 0$ and $b^2 - 4ac = -31 < 0 \Rightarrow 2x^2 - 3x + 5 > 0$

for all $x. \Rightarrow 2x^2 - 3x + 5 < 0$ does not have any solutions.

$\Rightarrow$ Hence the solution set $= \phi$.

29. The inequation is $2x^2 - 3x + 10 > 0$

Now $a = 2 > 0$ and $b^2 - 4ac = -71 < 0 \Rightarrow 2x^2 - 3x + 10 > 0$ for all x

$\Rightarrow$ The solution set $x \in R$

30.(C) $(x-2)^2 - 3|x-2| + 2 = 0$

$$|x-2| = 1, 2$$

$$|x-2| = 1 \Rightarrow x = 1, 3$$

$$|x-2| = 2 \Rightarrow x = 0, 4$$

Product of solutions = 0